
OS
LVSIADAS
de Luis de Camões.

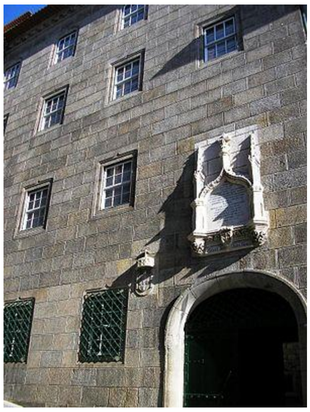
COM PRIVILEGIO
REAL.

Impressos em Lisboa, com licença da
sancta Inquisição, & do Ordina-
rio: em casa de Antonio
Gõçaluez Impressor.
1572.

From Henry the Navigator over Os Lusiadas to a Europe in Change

INTRODUCTION

Who was Henry the Navigator?



The 5th son of King João I (John I) and his wife D. Filipa de Lencastre (Philippa of Lancaster), named “The Navigator”, was born in Porto on 4 March, 1394, in the place called today, “Casa do Infante”. Today it is possible to visit the Interpretive Center "Infante D. Henrique and the New Worlds" there. He died in Sagres on 13 November, 1460. His remains are currently in the Monastery of Batalha, built by his father.

He dedicated himself to the study of Mathematics, and especially to Cosmography. He was the commander of the fleet that left Porto for the conquest of Ceuta (1415), a city located in the North of Africa, near the Strait of Gibraltar. This location gave it an important strategic position in the Mediterranean trade domain, besides also being an important commercial warehouse between Asia, Africa and Europe.

In 1416 or 1419 he founded a school of cosmography and navigation on the promontory of Sagres. He also established shipbuilding yards and workshops and erected the first Portuguese astronomical observatory. With the necessary conditions in place, the discovery of the West African coast by sea began.

What is Os Lusiadas?

Os Lusiadas is a book written by Luís de Camões, born in Lisbon in 1524 (?) and deceased in the same place on 10 June, 1580. He is buried in the Jerónimos Monastery.

It is the greatest work of Portuguese epic poetry consisting of ten parts called "cantos". It is thought to have been completed in 1556 and published for the first time in 1572, three years after the return of Luís de Camões from East Asia.

The main theme of the work is the Portuguese discoveries and the sea way to India, although we can also find other episodes about the history of Portugal, in which the main hero is the Portuguese people.

Context

This is the Portuguese theme. Our Portuguese partner school is a music school and Henry the Navigator is known for his support to Portuguese seafarers during the Age of Discoveries. That’s why this set of lessons will take you on a journey through Europe following the sounds of the monochord and inspiring you to a poem, a song, a rap, a theatre play as was also done in Os Lusiadas. Just let your creativity run wild!

PART 1

Topic	What is sound and which quantities are important?
Subjects	Science, Technology
Level	★ ★
Aims	The students learn about sound and its quantities.
Skills	Science: learning about wave lengths, speed of sound, frequency and the influence of temperature Technology: using an app to experiment with different quantities of sound
Duration	100’
Resources	Computer or mobile phone + app

SOUND

What is sound and how fast does it move?

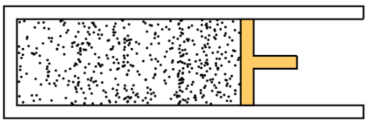
Watch this video.

http://seilias.gr/images/stories/myvideos/senaria/senario-5-ixos/soundFiles/sound1.mp4

What kind of movement do you cause to happen when playing an instrument?

Sound is nothing more than vibrating air. The simulation underneath shows you how air molecules move in a sealed off area and at a particular temperature.

http://seilias.gr/erasmus/html5/gas.html

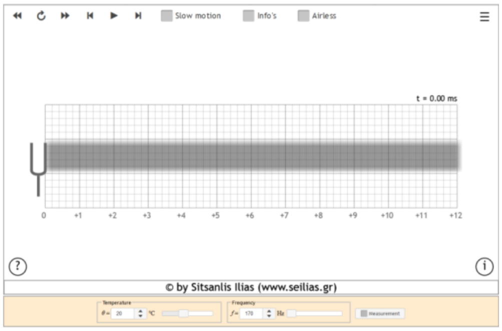


Enlarge and scale down the area and describe what happens.

Sound is a sequence of increases and decreases in air pressure. Or if we want to put it more scientifically: sound is every audible change of density in an elastic medium.

Click on the button ‘info’ in the simulation underneath and observe the changes in air pressure. Click on the button ‘airless’ and see what happens in a vacuum.

http://seilias.gr/erasmus/html5/soundSpeed.html



Activity 1

Let’s do some experiments with the simulation
Choose a temperature of 20°C and a frequency of 300 Hz.
How long does the sound travel approximately to cover a distance of 12 m?

What is the speed of the sound at 20°C?

And what is the speed of the sound at -40°C and at 100°C?

Temperature (°C)	Speed of sound(m/s)
20	
100	
-40	

What can we conclude?

A simplified formula that represents the relation between the speed of the sound and the temperature in °C is $v = 20\ x\ \sqrt{273 + T}$, with v the speed of sound measured in m/s and T the temperature in °C.

Use a calculator to check if your experiment was successful.

Once more choose a temperature of 20°C. We change the frequency from 100 Hz to 500 Hz. Select the button ‘measurement’. On the screen you see the moment on which the wave passes the first and the second microphone.

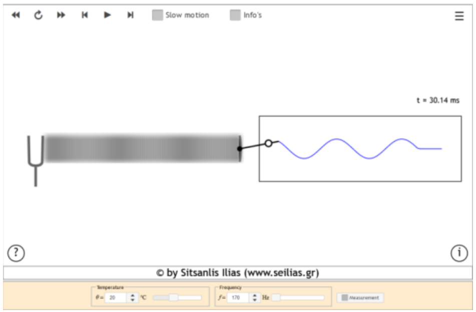
What can we conclude?

The importance of the frequency of sound

Sound travels under the form of changes in density of a medium and moves as a longitudinal wave. This is a wave that moves in the same direction as the propagation of the energy.

You can see this wave in het simulation underneath.

http://seilias.gr/erasmus/html5/waveForm.html



Activity 2

Let's examine the influence of the frequency on a sound.

Once more choose a temperature of 20 °C and a frequency of 170 Hz. Calculate the speed of the sound like in activity 1.

What is the speed?

The wave appears at the moment the sound reaches the end position. Freeze the simulation and note the time(t_1). Unfreeze the simulation till the wave arrives at its highest point. Freeze the simulation again and note the time(t_2). Unfreeze the simulation again till the waves arrives at the next highest point. Freeze the simulation again and note the time (t_3) Do the same for the frequencies 200 Hz and 300 Hz.

Frequency(Hz)	t_1(s)	t_2(s)	t_3(s)	$t_3 - t_2$(s)
170				
200				
300				

The frequency has no influence on

The frequency has an influence on

At a frequency of 170 Hz you will observe a high and low pressure times per second.

At a frequency of 200 Hz you will observe a high and low pressure times per second.

At a frequency of 300 Hz you will observe a high and low pressure times per second.

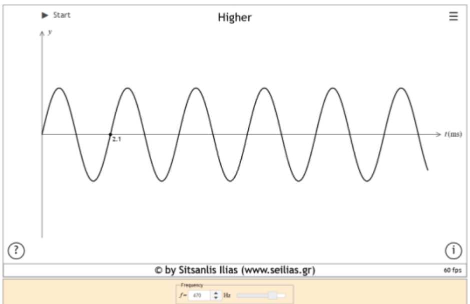
The formula $v = \lambda \times f$ allows you to calculate the wave length (λ in m).

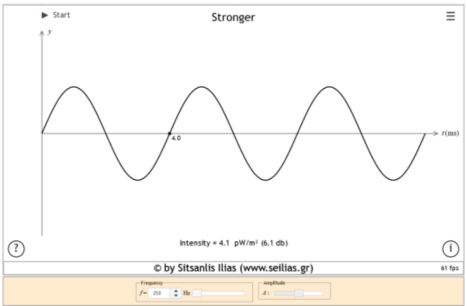
Speed of sound v (m/s)	Frequency f(Hz)	Wavelenght λ (m)
	170	
	200	
	300	

You can clearly hear the influence of the frequency on the wave length in the first simulation. In the second one you can hear what happens when you change the height of the wave. The height of a wave is called its amplitude.

http://seilias.gr/erasmus/html5/sound-1.html

http://seilias.gr/erasmus/html5/sound-2.html





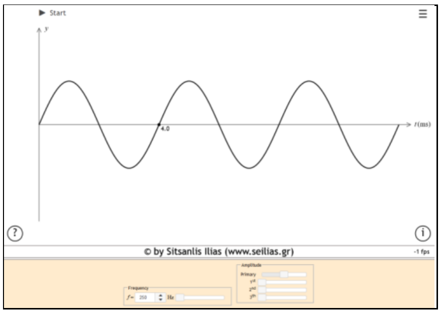
Conclusion:

The larger the frequency , the   the tone of the sound.

The larger the amplitude, the   the sound.

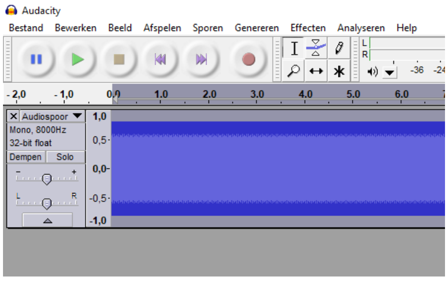
The loudness of a sound is called the sound intensity. The sound intensity level is measured in decibel (dB). The frequency of a sound is called the pitch and is measured in Hz.

http://seilias.gr/erasmus/html5/sound-3.html



Activity 3

Download the program Audacity.



Some particularities. If you don’t know the answer, look it up on the internet.

Open the program Audacity and click on ‘generate’. Next choose ‘tone’.

What is the smallest frequency that you can hear?

What is the largest?

What frequencies of sound should normally be perceivable by the human ear.

Which intensity of sound can damage your hearing?

You hear the thunder 10 seconds later than seeing the lightning. How far away is the thunderstorm?

The lowest tone of a piano has a wavelength of 12.5 m, the highest tone one of 8.2 cm.

What range of frequencies does a piano cover?

PART 2	
Topic	Students make their own monochord
Subjects	Technology, Engineering, Art, Math
Level	★ ★
Aims	Students learn about the different scales that through time were used to understand musical notes and their relation.
Skills	Technology: using apps to see and hear these differences Engineering: making your own musical instrument, a monochord Art: studying the basic material for music, notes and octaves Mathematics: using fractions to discover and describe intervals between notes
Duration	200’
Resources	3 plywood boards of multiplex plates of 65 cm x 10 cm x 1.8 cm, a stick of 0.5 cm x 0.5 cm x 85 cm, a guitar string, some nails, glue and a screw or something of the kind.

INTRODUCTION

Now that we know which quantities play a part in producing sound, we can start making our own musical instrument: the monochord.

This is a string instrument with one string and a sound box. Underneath the string you need to be able to move a bridge along a scale. But what happens when you pluck a string?

We provide you with an app (Piano.apk) that allows you to hear the different notes as they are played on this instrument.

THE MONOCHORD, WAVES AND SCALES

The monochord was used by the ancient Greeks in order to measure and check a scale. Pythagoras is believed to have invented the instrument while examining the ratio between two sounds.

When he used the entire string, the note DO was heard, as can be seen in figure *2A*. When he made only half a string vibrate, he obtained the DO from a higher octave, *Fig.2B*. The frequency ratio between the DO of the higher octave and the other DO is 2:1.

Fig.2A

Fig.2B

We call this an octave because this is the distance between the first note and the subsequent eighth note.

When he moved the bridge such that the string was divided into three equal parts, making only two parts of the string vibrate, (Fig. 3B), he obtained a perfect fifth, SOL. The frequency ratio between this sound and the DO is 3:2.

Fig.3A

Fig.3B

We call this a perfect fifth because it is the distance between the DO and the subsequent fifth note, SOL.

In order to understand the Pythagorean scale we are going to learn a little about expansion, reduction, addition and subtraction of intervals

The intervals between two sounds can be represented by fractions.

For example, the interval between the frequency sounds f = 600 Hz and f '= 400 Hz is represented by the fraction $\frac{3}{2}$, since $\frac{f}{f\prime} = \frac{600}{400} = \frac{3}{2}$. It means that between the frequency 600 Hz and the frequency 400 Hz there is a ratio of 3:2. It is usual to make the quotient between the highest frequency sound and the lowest frequency sound.

The interval between the DO and the SOL is a perfect fifth.
With two sounds that form an interval of an octave, their frequencies will be in the ratio of 2:1 or $\frac{2}{1}$.

Increasing the range of acoustic intervals by one octave

To increase an interval we have to increase the frequency of the highest sound or to decrease the frequency of the lower sound.

Example - Considering the interval $\frac{25}{24}$, we calculate the interval corresponding to the increase of an octave of this interval.

To increase by one octave, we can reduce the frequency of the lower sound by half, that is, we would have $\frac{25}{\frac{24}{2}} = \frac{25}{12}$ or we could increase the frequency of the higher sound to its double and we would have $\frac{2\,x\,25}{24} = \frac{25}{12}$.

Conclusion: Increasing the range by one octave corresponds to doubling the frequency ratio.

Reducing the range of acoustic intervals by one octave

The reduction of intervals has a similar reasoning as that of increasing intervals, but in the opposite direction.

Example - Considering the interval $\frac{32}{15}$, we calculate the interval corresponding to the decrease of that interval by one octave.

To reduce the range by one octave, we can double the frequency of the lower sound, so we would have $\frac{32}{15\,x2} = \frac{32}{30} = \frac{16}{15}$ or we could also lower the frequency of the higher sound by half and we would have $\frac{\frac{32}{2}}{15} = \frac{32}{30} = \frac{16}{15}$

Conclusion: Decreasing the interval by one octave corresponds to halving the frequency ratio.

Calculating other acoustic intervals

We try to calculate the interval from DO to SI, knowing that:

Interval DO-LA = $\frac{5}{3}$ and interval LA - SI = $\frac{9}{8}$

DO-SI = (DO-LA) + (LA-SI)

$\frac{f(SI)}{f(DO)} = \frac{f(LA)}{f(DO)}\, x\, \frac{f(SI)}{f(LA)} = \frac{5}{3}$ x $\frac{9}{8} = \frac{45}{24} = \frac{15}{8}$

Conclusion: The addition of intervals corresponds to the multiplication of the frequency ratios.

We try to determine the interval from FA to SOL, knowing that:

Interval DO-FA = $\frac{4}{3}$ and interval DO-SOL = $\frac{3}{2}$

FA - SOL = (DO - SOL) - (DO-FA)

$$\frac{f(SOL)}{f(FA)} = \frac{\frac{f(SOL)}{f(DO)}}{\frac{f(FA)}{f(DO)}} = \frac{\frac{3}{2}}{\frac{4}{3}} = \frac{9}{8}$$

Conclusion: The subtraction of intervals corresponds to the division of the frequency ratios.

PYTHAGOREAN SCALE

After understanding the expansion and reduction of intervals we can understand how the Pythagorean scale emerges.

To build his scale Pythagoras used the sounds obtained through successively shortening vibrating strings with 1/3 of the length of the string (*Figure 4).* The sound obtained by each of the new strings obtained by successive reduction of a 1 / 3 of the string length is at a perfect fifth interval with respect to the sound of the previous vibrating string.

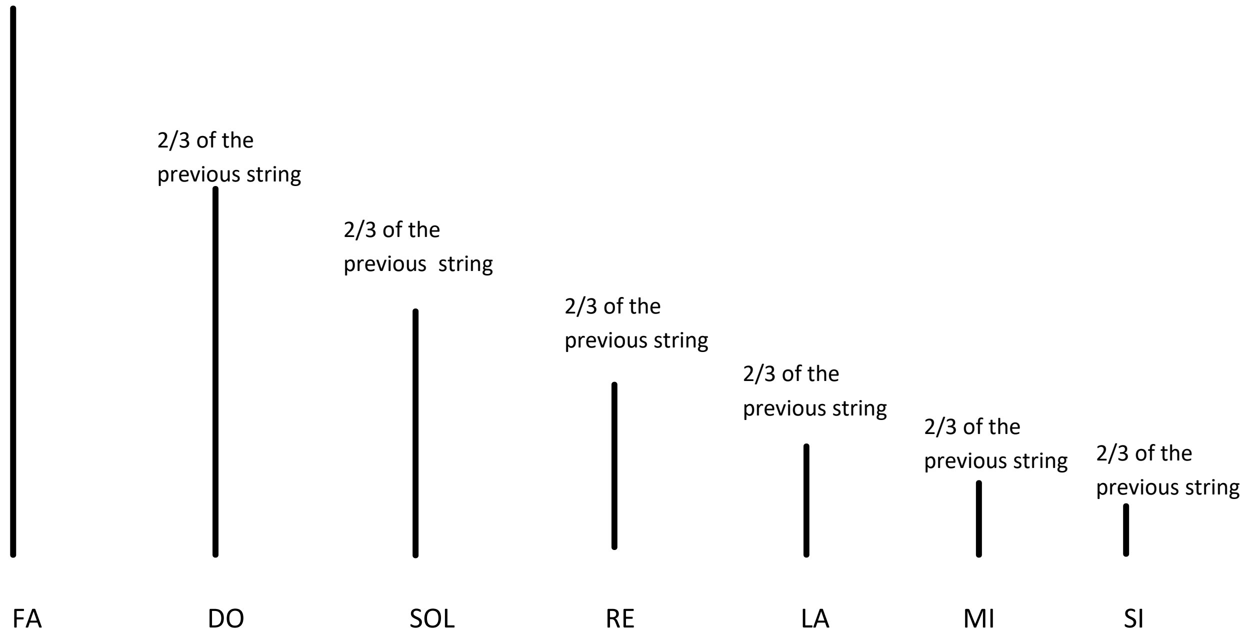


Fig.4

DO-RE acoustic interval

For instance, the interval between DO and RE represents two perfect fifths and so will be:
3/2 x 3/2 = 9/4 (adding intervals = multiplying frequency ratios)
Since the interval between this DO and RE exceeds the octave it is necessary to lower the RE by one octave and therefore:
Divide 9/4 by 2 = 9**/8 which is the value of the Pythagorean tone**.
The DO-RE acoustic interval ratio is 9/8

DO-MI acoustic interval

For the interval between DO and MI:

FA **DO** SOL RE LA **MI** SI

3/2 x 3/2 x 3/2 x 3/2 = 81/16

Since the MI is two octaves higher it must be reduced by two octaves.
For this it is necessary to divide 81/16 by 4 = **81/64**
The DO-MI acoustic interval ratio is 81/64

MI-FA interval

For the interval between MI and FA:

FA **DO** SOL RE LA **MI** SI

3/2 x 3/2 x 3/2 x 3/2 x 3/2 = 243/32
To reduce the MI three octaves we will have to divide 243/32 by 8 and we will get 243/256.
This is actually the FA-MI interval by reduction.
For the Pythagorean ascending MI-FA we will have the inverse relation, hence 256/243.
The MI-FA acoustic interval ratio is 256/243.

This way we have just discovered The Pythagorean halftone (example MI to FA), which is 256/243 while the Pythagorean tone (example DO to RE) is 9/8.

We can check that in the scale of Pythagoras two half tones do not equal a tone.

Let's see:

2 half Pythagorean tones = 256/243 x 256/243 = 1.1099 and 1 Pythagorean tone = 9/8 = 1.125
The interval which is missing from two Pythagorean half-tones to be a Pythagorean tone is called the Pythagorean coma.

Pythagorean Coma = 1 Pythagorean tone - 2 half Pythagorean tones

Activity 4

Up to you to complete the table of the frequency ratio for each acoustic interval in the Pythagorean scale.

Table 1 - Frequency ratio for each acoustic interval						
DO-RE	RE-MI	MI-FA	FA-SOL	SOL-LA	LA-SI	SI-DO
9/8	9/8	256/243	9/8	9/8	9/8	256/243

In Table 2 we represent the frequency ratios between each note and the tonic (DO).

Using the information given in Table 1, it is now up to you to calculate the frequency ratio between DO-LA and DO-SI.

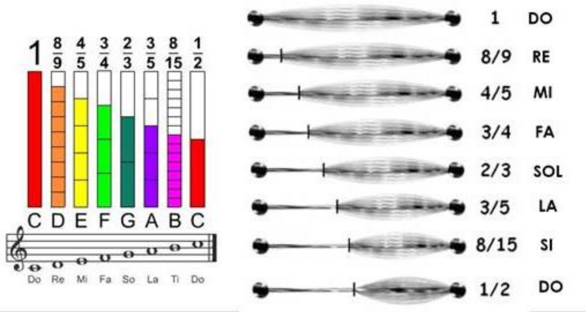
Under a constant tension the length of the string is inversely proportional to the height of the sound. Complete table 2 with the length of the string.

Table 2 – Pythagorean scale								
Musical notes	DO	RE	MI	FA	SOL	LA	SI	DO
Frequency ratio between each note and the tonic (DO)	1/1	9/8	81/64	4/3	3/2	27/16	243/128	2/1
Length of the string								

ZARLINO SCALE (NATURAL TUNING)

Pythagoras’ system is perfect for music that was done in Antiquity and Middle Ages but it isn’t for polyphonic music, since it can’t be used for chords. The **Zarlino scale (natural tuning)** appears. For as long as 300 BC people were thinking of alternatives to Pythagoras´ tuning.

The alternative underneath uses simpler fractions.



Activity 5

In Table 3 we present the frequency ratio for each acoustic interval in the Zarlino scale.

Table 3 - **Zarlino scale (natural tuning)**						
Frequency ratio for each acoustic interval						
DO-RE	RE-MI	MI-FA	FA-SOL	SOL-LA	LA-SI	SI-DO
9/8	10/9	16/15	9/8	10/9	9/8	16/15

We notice that in Zarlino's scale we have three different intervals (9/8,10/9 and 16/15).

How many did we have in the Pythagorean scale?

In Table 4 we represent the frequency ratios between each note and the tonic in the Zarlino scale.

Using the information given in Table 3, it is up to you to calculate the frequency ratio between DO-MI and DO-FA.

Table 4 - **Zarlino scale (natural tuning)**								
Musical notes	DO	RE	MI	FA	SOL	LA	SI	DO
Frequency ratio between each note and the tonic	1	9/8	5/4	4/3	3/2	5/3	15/8	2

EQUAL TEMPERAMENT TUNING

The Pythagorean scale and the Zarlino scale (pure tuning) present some practical difficulties, especially for tuning keyboards, because they contain unequal frequency ratios.

To solve this problem they created the equal temperament tuning that was made up to contain 12 notes separated by equal intervals. The interval between two subsequent notes is always $\sqrt[12]{2}$.

The notes are: DO, DO#, RE, RE#, MI, FA , FA#, SOL, SOL#, LA, LA#, SI *(Fig.6).*

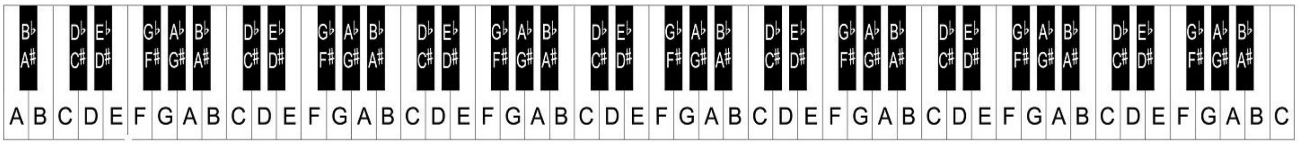


Fig.6

To find the frequency of a note, f_n, knowing the preceding note, f_{n-1}, the following expression is used:

$$f_n = f_{n-1}\sqrt[12]{2} = 1.0595 \times f_{n-1}$$

In *Table 5* we represent the frequency ratio for each interval in equal temperament tuning.

Table 5 . **Equal temperament tuning**						
Frequency ratio for each acoustic interval						
DO-RE	RE-MI	MI-FA	FA-SOL	SOL-LA	LA-SI	SI-DO
$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^1$	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^1$

In *Table 6* we present the frequency ratio between each note and the tonic in an equal temperament tuning.

Table 6 – Equal temperament tuning								
Musical notes	DO	RE	MI	FA	SOL	LA	SI	DO
Frequency ratio between each note and the tonic	1	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^4$	$(\sqrt[12]{2})^5$	$(\sqrt[12]{2})^7$	$(\sqrt[12]{2})^9$	$(\sqrt[12]{2})^1$	2

Activity 6

But what is the use of all that math?

The math will allow you to complete the frequency of all the different notes of all scales.

With these fractions we can obtain the frequency of the different notes in each scale, knowing the frequency of one of the notes (for instance LA 440 Hz).

Try it!

Musical Notes	**Pythagorean scale**	**Perfect tuning**	**Equal temperament tuning**
DO	261	264	261.6
RE	294	297	293.7
MI	330	330	329.6
FA	348	352	349.2
SOL	392	396	392
LA	440	440	440
SI	495	495	493.9

Let's take the fourth octave as an example. The simulations underneath allow you to listen and try to distinguish between the different tunings of for example the musical note FA.

http://seilias.gr/erasmus/html5/notesPythagora.html

http://seilias.gr/erasmus/html5/notesReine.html

http://seilias.gr/erasmus/html5/notesAccuracy.html

Activity 7

Under a constant tension the length of the string is inversely proportional to the pitch of the sound.

So, if the entire string represents the 264 Hz DO, plucking the string with the bridge halfway, the sound will be the 528 Hz DO.

Complete the tables below with the length of the string.

Which of the tunings underneath will be the easiest in order to build our monochord as accurately as possible.

Why?

Pythagorean scale	**C do**	**D re**	**E mi**	**F fa**	**G sol**	**A la**	**B si**	**C do**
Frequency ratio between each note and tonic	1	9/8	81/64	4/3	3/2	27/16	243/128	2
Length of string *(m)*	1							1/2

Zerlino scale	**do**	**re**	**mi**	**fa**	**sol**	**la**	**si**	**do**
Frequency ratio between each note and tonic	1	9/8	5/4	4/3	3/2	5/3	15/8	2
Length of string *(m)*	1							1/2

Equal temperament scale	**do**	**re**	**mi**	**fa**	**sol**	**la**	**si**	**do**
Frequency ratio between each note and tonic	1	$(\sqrt[12]{2})^2$	$(\sqrt[12]{2})^4$	$(\sqrt[12]{2})^5$	$(\sqrt[12]{2})^7$	$(\sqrt[12]{2})^9$	$(\sqrt[12]{2})^{11}$	2
Length of string *(m)*	1							1/2

Activity 8

Let's make our monochord.

What you need: 3 plywood boards of multiplex plates of 65 cm x 10 cm x 1.8 cm, a stick of 0.5 cm x 0.5 cm x 85 cm, a guitar string, some nails, glue and a screw or something of the kind.

You will need to accurately shorten the 60 cm string to play a specific note. You can do this by putting a bridge under the string. Therefore you need to calculate and indicate the correct position of this bridge for each note. You will find those positions by recalculating all fractions underneath to fractions with denominator 60.

Complete table 11

Table 11								
Scale	DO	RE	MI	FA	SOL	LA	SI	DO
String length fraction	1	8/9	4/5	3/4	2/3	3/5	8/15	1/2
Marks on the string of 60 cm								

Take the stick and saw off 2 pieces of 10 cm and 1 piece of 60 cm.

Mark the centimeters on the 60 cm piece and highlight the pure tuning spots.

So when we take the 60 cm string and put the bridge on these highlighted markings in order to shorten the string, we will subsequently hear the notes DO, RE, MI, FA, …

If possible use a milling cutter to remove 1.8 cm x 65 cm x 0.5 cm of wood from the sides of one of the boards.

Assemble all parts as shown in the picture underneath. On one side the string needs to be attached to a nail, on the other side you use a screw, or something likewise.

Put tension on the string by turning the screw. Be careful not to break it.

Also make sure the string rests on the stick near the screw.

Use an object as a bridge to shorten the length according to the marked spots and play

Ultimately this might not give you a perfect result.

Watch this simulation: http://seilias.gr/erasmus/html5/notesReine.html

What is the frequency of the DO in the different octaves?

	1st	2nd	3rd	4th	5th	6th	7th
Frequency DO(Hz)	33	66	132	264	528	1056	2112

An ideal string produces vibrations corresponding to the formula $f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$

With n = 1,2,3,4… , f the frequency in Hertz, L the length of the string in metre, T the tension of the string in Newton and μ the mass per metre in kg/m.

Consequently not only the length of the string is important. Also the mass and tension play a part.

Activity 9

And for the musicians amongst us, they can try this song.

PART 3	
Topic	Travelling on the notes of our monochord and telling the story of your travel
Subjects	Science – Technology - Art
Level	★
Aims	Students discover Europe in a playful way, its history, its traditions, its qualities, … by creating their own trip on the notes of the monochord.
Skills	Science: making a geographical journey through countries and cities Technology: using Google Earth to create a journey Art: Creating a song, a poem, a theatre play ….. describing their journey
Duration	150 min
Resources	Google Earth - Internet

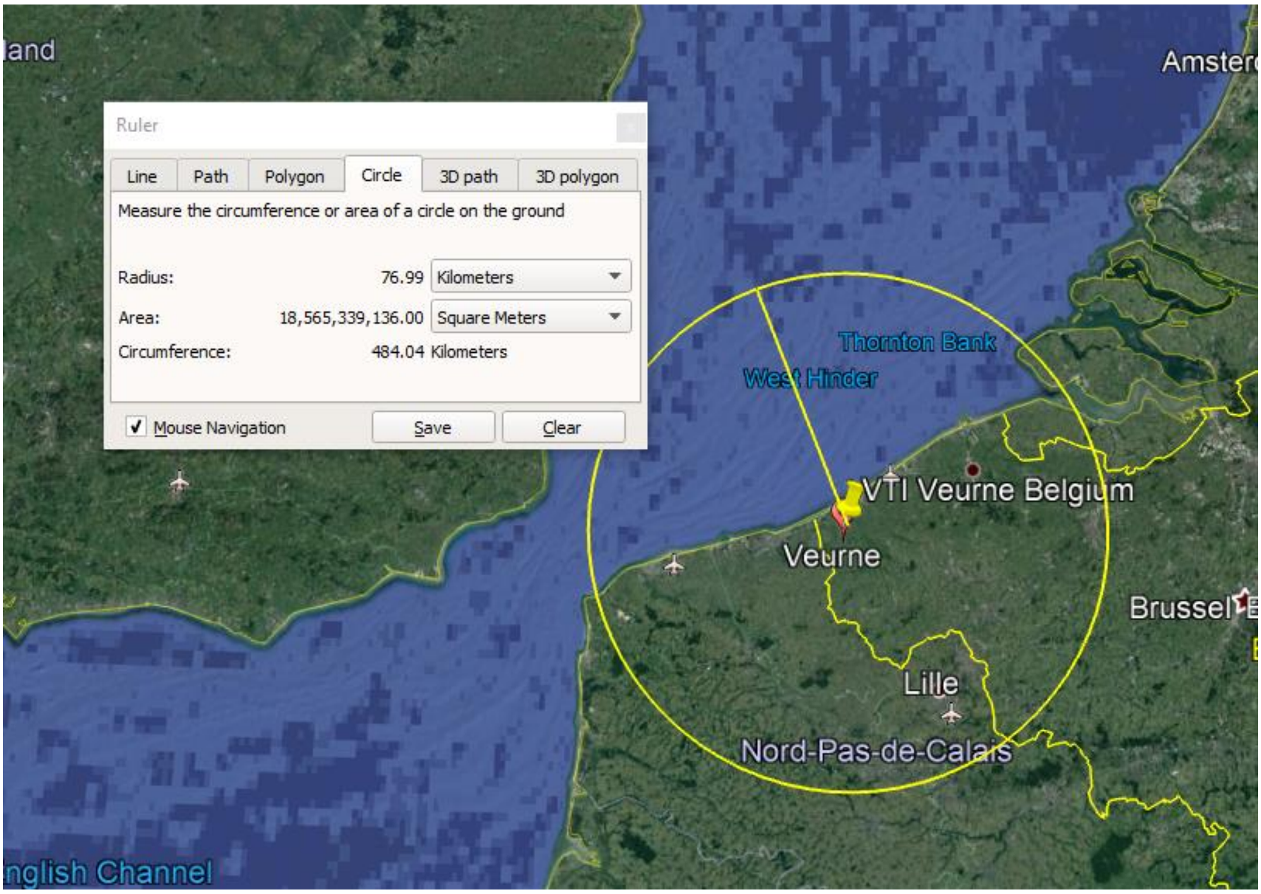
Activity 10

Put the fractions of p 21 on a denominator 1000.

DO	1000/1000
RE	
MI	
FA	
SOL	
LA	
SI	
DO	

For this activity we use Google Earth.

Locate your hometown on Google Earth. Take the Ruler tool and choose the Circle. Draw a circle around your town.



Zoom out so that you can see all of Europe. Increase the circle until the radius is equal to the nominator of the note 'RE' in km. Choose a country situated on the circumference of the circle. Zoom in on that country and choose a city that you know and is situated within the area of the circle.

Now we make a virtual trip to that city on the internet. Maybe you will find someone famous who was born there, maybe you will find a delicious meal, maybe famous inventors worked there, maybe something interesting was invented there or maybe … . Search the internet and fill in the table underneath.

From this place you continue your journey in the same way, by drawing a circle with a radius equal to the nominator of MI in km. But you need to stay in Europe.

Now fully complete this table.

	Country	City	Point of interest
DO			
RE			
MI			
FA			
SOL			
LA			
SI			
DO			

Activity 11

Write a poem, a song, a theatre play or make a drawing or painting that describes your journey. Finally present your trip to the class.

In case you have no inspiration, this is what you could do: write a short haiku poem.

Haiku is a traditional Japanese poetry form. It is a way of looking at the physical world and seeing something deeper, like the very nature of existence. It should leave the reader with a strong feeling or impression.

A haiku uses just a few words to capture a moment. A haiku is written in three lines, with five syllables in the first line, seven syllables in the second line, and five syllables in the third line. The lines rarely rhyme.

Perhaps you can first check out a couple of haikus on the internet.

Now brainstorm words that are about your topic. Write your three-line poem with the pattern 5-7-5.